Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year **(تخلفات)**		Final Term Exam Date: December 25 , 2019 Course: Mathematics 1 – A Duration: **3** hours

The Exam consists of one page Answer **All** Questions No. of questions: 4 Total Mark: 100

<u>Question 1</u>	
(a) If $A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 & 1 \\ 1 & -3 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 2 \\ 1 & -3 & 1 \end{bmatrix}$	10
Find, if possible $A + B$, $A + C$, $A.B$, $A.C$, $\lvert A\rvert$, $\lvert C\rvert$, A^{-1}, C^{-1}.	
(b)Find the eigenvalues and the eigenvectors of the matrix : $A = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$	10
Also, Write the Hamilton equation, the diagonal form and find $f(A) = 2^A$.	
(c)Solve the L.S : (i) $3x - y + z = 3,\quad 2x + 2z = 6,\quad -x + y + z = 2$	5
(ii) $2x - 2y + z = 2,\quad -2x + 3y - z = 0,\quad 2z - y + 3x = 8$	
<u>Question 2</u>	
(a)By Induction, prove that : $1 + 2 + 3 + \cdots + n < \frac{(2n+1)^2}{8},\ n \geq 1$	25
(b)Expand $\frac{x-3}{(x-1)(x-2)}$	
(c)If $z_1 = 1 + i$, $z_2 = -8i$. Find $(z_1.z_2)^4$ $(z_1/z_2)^5$ and $\sqrt[6]{z_2}$.	
(d)Find S_n , S_{100} and S_∞ from the series : $\sum_{r=1}^{n} \frac{1}{(3r-1)(3r+2)}$	
(e)Find the coefficient of y^5 in the expansion $(y^2 + \frac{1}{y})^7$.	
<u>Question 3</u>	
3a)Find the first derivative for: $y = \cot(\sec(x^2)) + \csc(e^{4x}) \tan(x) + e^{x^2}$	9
3b)Evaluate the limits i) $\lim\limits_{x \to 0} \frac{1 - \cos x}{x^2}$, ii) $\lim\limits_{x \to 1} [\frac{1}{\ell n\, x} - \frac{1}{x-1}]$	6
3c)Find the n^{th} derivative of: (i) $f(x) = \cos(5x) \sin(2x)$ ii) $g(x) = 5^{3x}$	6
3d) $f(x) = x^2 \cos x$, expand in Taylor Maclaurin.	5
<u>Question 4</u>	
4a) Evaluate the following integrals	12
i) $\int \frac{dx}{\sqrt{(2x - 3x^2 + 4)^3}}$ ii) $\int x^3 \sin(4x)\, dx$ iii) $\int \frac{(x-2)dx}{(x^2 - 4x + 3)^3}$	
iv) $\int \frac{dx}{(x^2 + 4x + 5)^2}$ v) $\int \frac{x^2 + 5}{x^2 + 3x + 2} dx$ vi) $\int [1 + \frac{1}{x}] \cos(x + \ln x)\, dx$	
4b) If $x = 5t^3$ and $y = 4t^2$ at time t, find the magnitude and direction of the velocity when $t = 10$	4
4c) A stone is dropped into a pond, the ripples forming concentric circles which expand. At what rate is the area of one of these circles increasing when the radius is 4 m and increasing at rate of 0.5 m / s?	4
4d) Determine all the numbers c which satisfies the conclusions of the mean value theorem for the function $f(x) = x^3 + 2x^2 - x$ on [-1, 2]	4

Good Luck *Dr. Reda Abd Ekader* *Dr. Khaled Elnagar*

Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year		Final Term Exam Course: Mathematics 1 – A Date: January 5 , 2019 Duration: **3** hours
The Exam consists of one page	Answer **All** Questions	No. of questions: 4 Total Mark: 100

Question 1

(a)Complete the following : 5

(i)The sum of all eigenvalues of a unit matrix of order n equals to……………………..

(ii)A square matrix A is called non singular if …….and it is called symmetric if ……

(iii)If λ is eigenvalue of a matrix A , then $\lambda + \sqrt{\lambda}$ is eigenvalue to ………….……..

(iv)A linear system AX = B is called inconsistent if ……………………………………

(v)A linear system AX = B has one solution if ………………………………………

(b)If $A = \begin{bmatrix} 0 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & 1 \\ 3 & -3 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}$ 8

Find, if possible A + B , A + C, A.B , A.C , C.A , |A| , |C|

(c)Find the eigenvalues and the eigenvectors of the matrix : $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ 8

Also, Write the diagonal form and find $f(A) = \ln A$.

(d)Solve the L.S : (i) $x - 2y + z = 1, \quad -2x + 3y - z = 0, \quad 3x + y - 2z = -3$ 2

(ii) $x - y + 2z = 2, \quad -x + y - z = 3, \quad 3x - 3y + 4z = 3$ 2

Question 2

(a)Find the coefficient of y^4 in the expansion of $(y^2 + \frac{1}{y^2})^6$ (b)Expand $\frac{2x-3}{(x-1)(x-2)}$ 3+3

(c)If $z_1 = 2 + 2i$, $z_2 = 8i$. Find $(z_1.z_2)^6$, $(\frac{z_2}{z_1})^{10}$ and $\sqrt[4]{z_2}$. 4

(d)Find u and v of the function $f(z) = \cos(iz)$. 2

(e)Solve $4x^3 - 24x^2 + 48x - 32 = 0$ if its roots a, b, c form a geometric sequence. 3

(f)Find S_n , S_{100} and S_∞ for the series : $\sum_{r=1}^{n} \frac{1}{(3r-1)(3r+2)}$ 5

(g)By Induction, prove that : $9^n - 2^n$ is divisible by 7, $n \geq 1$. 5

Question 3

(a)Find $\frac{dy}{dx}$ where : (i) $y = 3\sin^3 t, \quad x = 2\cos^3 t$ (ii) $e^x \sin y = e^y \cos x$ 12

(iii) $y = e^{5x^2}.(\operatorname{arcsec} x)^8.(7 - \operatorname{csch} 3x)^{-12}$

(b)Find $y^{(18)}$ where $y = \frac{3-5x}{6x+8}$. 5

(c)Find the integral : $\int (8^{3x} + x^2 e^{-5x^3} - \cot 4x + \frac{x^3}{\sqrt{x-5}})dx$ 8

Question 4

(a)If the hypotenuse of right triangle is constant. Show that its area is maximum when it is isosceles . 5

(b)Find the equation of tangent and normal for the curve: $y = \frac{(\ln x)^x}{2^{\ln x - 1}}$ at x = e. 8

(c)Find the limits: (i) $\lim\limits_{x\to\infty} \frac{5^x + 2^x}{9^x}$ (ii) $\lim\limits_{x\to\infty} x^{\frac{1}{\sqrt{x}}}$ (iii) $\lim\limits_{x\to\infty} e^{-x} \ln x$ (ii) $\lim\limits_{x\to 0}(\frac{1}{x} - \frac{1}{e^x - 1})$	12

Good Luck *Dr. Khaled Elnagar* *Dr. Mohamed Eid* *Dr. Elsayed Gareeb*

Model Answer

Answer of Question 1

(a)(i)The sum of all eigenvalues of a unit matrix of order n equals to n.

(ii)A square matrix A is called non singular if $|A| \neq 0$ if and it is called symmetric if $A = A`$

(iii)If λ is eigenvalue of a matrix A , then $\lambda + \sqrt{\lambda}$ is eigenvalue to $A + \sqrt{A}$.

(iv)A linear system AX = B is called inconsistent if it has no solution.

(v)A linear system AX = B has one solution if rank A = rank G = number of unknowns.

---*5 Marks*

(b)If $A = \begin{bmatrix} 0 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & 1 \\ 3 & -3 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}$

A + C, A.B, C.A and |A| are nor exist.

$A + B = \begin{bmatrix} 2 & 1 & 5 \\ 4 & 0 & 0 \end{bmatrix}$, $A.C = \begin{bmatrix} 4 & -4 & 10 \\ 7 & -2 & 6 \end{bmatrix}$, $|C| = 1 + 8 - 6 = 3$

---*8 Marks*

(c) $\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda) - 2 = \lambda^2 - 5\lambda + 4 = 0$. Then $\lambda_1 = 1$, $\lambda_2 = 4$

From the equation, $\begin{bmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

For : $\lambda_1 = 1$, $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then x + 2y = 0, x + 2y = 0

Put x = 2, then the corresponding

eigenvector is : $X_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

For : $\lambda_2 = 4$, $\begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then –2x + 2y = 0, x – y = 0

Put y = 1, we get x = 1 and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Then $T = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$ and $T^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$

The diagonal form : $T^{-1}A\,T = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$. Then $T^{-1} \ln A\; T = \begin{bmatrix} \ln 1 & 0 \\ 0 & \ln 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1.4 \end{bmatrix}$

Then $\ln A = T\begin{bmatrix} 0 & 0 \\ 0 & 1.4 \end{bmatrix}T^{-1} = \frac{1}{3}\begin{bmatrix} 1.4 & 2.8 \\ 1.4 & 2.8 \end{bmatrix}$

---*8 Marks*

(d)By Calculator, the solution is : (1, 2, 4).

(ii) $G = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ -1 & 1 & -1 & 3 \\ 3 & -3 & 4 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & -2 & -3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 7 \end{array}\right]$

Rank A = 2, Rank G = 3. No solution.

--*4 Marks*

Dr. Mohamed Eid

Answer of Question 4

a) Since the hypotenuse of the right triangle is constant, therefore $x^2 + y^2 = c^2 \Rightarrow$ $2x + 2yy` = 0 \Rightarrow y` = -x/y$

Area: $A = \frac{1}{2}xy \Rightarrow \frac{dA}{dx} = \frac{1}{2}(xy`+y) = \frac{1}{2}(x(\frac{-x}{y}) + y) = 0 \Rightarrow x^2 = y^2 \Rightarrow x = y$

Therefore the area is maximum when the triangle is isosceles.

b) $y` = \dfrac{[\ell n\, x]^x [\ell n(\ell n x) + x(\frac{1/x}{\ln x})][2^{\ell n(x)-1}] - [\ell n\, x]^x [\frac{1}{x}][2^{\ell n(x)-1}]\,\ell n\, 2}{[2^{\ell n(x)-1}]^2}$

at x = e, slope of the tangent = $y`(1) = 1 - \frac{\ell n 2}{e} = \frac{e - \ell n 2}{e}$

Therfore the point of contact is (e, 1) and hence the equation of tangent is $\frac{y-1}{x-e} = \frac{e - \ell n 2}{e}$

Also equation of normal is $\frac{y-1}{x-e} = \frac{e}{\ell n 2 - e}$

c-i) $\lim\limits_{x\to\infty} \frac{5^x + 2^x}{9^x} = \lim\limits_{x\to\infty} \frac{5^x(1 + (\frac{2}{5})^x)}{9^x} = \lim\limits_{x\to\infty} (\frac{5}{9})^x = 0$

ii) This limit of the indeterminate form ∞^0. Let $y = x^{1/\sqrt{x}} \Rightarrow \ell n\, y = \frac{\ell n\, x}{\sqrt{x}}$

Hence L'Hopital rule is ready to be used such that

$$\lim_{x\to\infty} \ell n\, y = \lim_{x\to\infty} \frac{\ell n\, x}{\sqrt{x}} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{1/x}{1/(2\sqrt{x})} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = 0\ .$$

Therefore $\lim\limits_{x\to\infty} x^{1/\sqrt{x}} = \lim\limits_{x\to\infty} e^{\ell n\, y} = e^{\lim\limits_{x\to\infty} \ell n\, y} = e^0 = 1$

iii) $\lim\limits_{x\to 0} [\frac{1}{x} - \frac{1}{e^x - 1}] = \lim\limits_{x\to 0} [\frac{e^x - 1 - x}{x(e^x - 1)}] = \lim\limits_{x\to 0} [\frac{e^x - 1}{xe^x + (e^x - 1)}] = \lim\limits_{x\to 0} [\frac{e^x}{xe^x + 2e^x}] = \frac{1}{2}$

iv) $\lim\limits_{x\to\infty} e^{-x}\, \ell n\, x = \lim\limits_{x\to\infty} \frac{\ell n\, x}{e^x} = \lim\limits_{x\to\infty} \frac{1}{xe^x} = 0$

Dr. Khaled EI Nagar

Final Exam and ILOs

Course Title: Mathematics 1-A **Code:** EMP 011

Questions	ILOs		
	Knowledge and Understanding	Intellectual Skills	Professional and Practical Skills
	a.1	b.1	c.1
Q1	√	√	√
Q2	√	√	
Q3	√		√
Q4	√	√	√

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